A Newtonian Model for the Quantum Gravitational Back-Reaction on Inflation

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Quantum gravitational back-reaction offers a simultaneous explanation for why the cosmological constant is so small and a natural model of inflation in which scalars play no role. In this talk I review previous work and present a simple model of the mechanism in which the induced stress tensor behaves like negative vacuum energy with a density proportional to $-\Lambda/8\pi G \cdot (G\Lambda)^2 \cdot Ht$. The model also highlights the essential role of causality in back-reaction.

The application of a force field in quantum field theory generally rearranges virtual quanta and thereby induces currents and/or stresses which modify the original force field. This is the phenomenon of *back-reaction*. Famous examples include the response of QED to a homogeneous electric field [1] and the response of generic matter theories to the gravitational field of a black hole [2]. In the former case, virtual e^+e^- pairs can acquire the energy needed to become real by tunnelling up and down the field lines. The newly created pairs are also accelerated in the electric field, which gives a current that reduces the original electric field. The event horizon of a black hole also causes particle creation when one member of a virtual pair passes out of causal contact with the other by entering the event horizon. As the resultant Hawking radiation carries away the black hole's mass the surface gravity rises.

Parker [3] was the first to realize that the expansion of spacetime can lead to the production of massless, minimally coupled scalars. Grishchuk [4] later showed that the same mechanism applies to gravitons. Production of these particles is especially efficient during inflation because of the causal horizon. Consider a spatially flat, locally de Sitter geometry with Hubble constant H, the invariant element for which is,

$$ds^2 = -dt^2 + e^{2Ht}d\vec{x} \cdot d\vec{x} . \qquad (1)$$

Whereas the co-moving time t corresponds to the duration of physical processes, the physical distance between two points $\vec{x}$ and $\vec{y}$ is not their coordinate separation, $\Delta\ell \equiv \|\vec{x} - \vec{y}\|$, but rather $e^{Ht}\Delta\ell$. This is a geometry in which Zeno's paradox about the impossibility of motion can come true! For if a photon is emitted at time $t = 0$ from an observer more distant than one Hubble length, H^{-1}, then the observer will never see it. By the time the photon has covered half the initial distance, the spacetime in between will have expanded by so much that the photon actually has *further* to go.

Now add the Uncertainty Principle. Even in flat space this requires virtual particles to be continually emerging from the vacuum and then disappearing back into it. But massless particles which are not also conformally invariant have a reasonable amplitude for appearing with wavelength greater than H^{-1}. When this happens in an inflating universe the particles must be ripped apart from one another by the Hubble flow. Since this creation mechanism requires both effective masslessness on the scale of inflation and the absence of conformal invariance, it is limited to gravitons and to light, minimally coupled scalars.

There is no doubt as to the reality of inflationary particle production because it is the usual explanation [5,6] for the primordial spectrum of cosmological density perturbations whose imprint on the cosmic microwave background has been so clearly imaged by the latest balloon experiments [7,8]. Our special interest is the back-reaction

from this process. There is no buildup of particle density because the 3-volume expands as new particles are created so as to keep the density constant. When a new pair is pulled out of the vacuum the one before it is, on average, already in another Hubble volume. However, the gravitational field is another thing. The created particles are highly infrared so they do not carry much stress-energy, but they do carry some, and this must engender a gravitational field in the region between them. Because gravity is attractive this field acts to resist the Hubble flow. Further, it is cumulative. The gravitational field of a created pair must remain behind to add with that of subsequent pairs, just as an observer continues to feel the gravity of a pebble even after dropping it into a black hole.

We believe that the gravitational attraction between virtual infrared gravitons gradually builds up a restoring force that impedes further inflation [9]. Gravity is a weak interaction, even at the enormous energies usually conceived for inflation, so the increment from each new pair is minuscule. But the effect must continue to grow until it becomes non-perturbatively strong, unless something else supervene to end inflation first. This mechanism offers the dazzling prospect of simultaneously resolving the (old) problem of the cosmological constant [10] and providing a natural model of inflation in which there is no scalar inflaton. The idea is that the actual cosmological constant is not small and that this is what caused inflation during the early universe. Back-reaction plays the crucial role of ending inflation.

A sometimes confusing point is that one does not require the complete theory of quantum gravity in order to study an infrared process such as this. As long as spurious time dependence is not injected through the ultraviolet regularization, the late time back-reaction is dominated by ultraviolet finite, nonlocal terms whose form is entirely controlled by the low energy limiting theory. This theory must be general relativity,

$$\mathcal{L} = \frac{1}{16\pi G}(R - 2\Lambda)\sqrt{-g} \,, \tag{2}$$

with the possible addition of some light scalars. Here "light" means massless with respect to $H \equiv$ $\sqrt{\Lambda/3}$. No other quanta can contribute effectively in this regime.

It is worth commenting that infrared phenomena can always be studied using the low energy effective theory. This is why Bloch and Nordsieck [11] were able to resolve the infrared problem of QED before the theory's renormalizability was suspected. It is also why Weinberg [12] was able to achieve a similar resolution for $\Lambda = 0$ quantum general relativity. And it is why Feinberg and Sucher [13] were able to compute the long range force due to neutrino exchange using Fermi theory. More recently Donoghue [14] has been working along the same lines for $\Lambda = 0$ quantum gravity.

To see a crude, Newtonian version of the process, consider locally de Sitter inflation on the manifold $T^3 \times R$, so that the physical radius of the universe at co-moving time t is,

$$r(t) \sim H^{-1}e^{Ht} \,. \tag{3}$$

The *bare* energy density of inflationally produced infrared gravitons — just the $\hbar\omega$ per graviton — is [4],

$$\rho_{\rm IR} \sim H^4 \,. \tag{4}$$

This is insignificant compared with the energy density of the cosmological constant, $\Lambda/8\pi G$, and $\rho_{\rm IR}$ is in any case positive. However, the gravitational interaction energy is negative, and it can be enormous if there is contact between a large enough fraction of the total mass of infrared gravitons,

$$M(t) \sim r^3(t)\rho_{\rm IR} \sim He^{3Ht} \,. \tag{5}$$

For example, if $M(t)$ was *all* in contact with itself the Newtonian interaction energy would be,

$$-\frac{GM^2(t)}{r(t)} \sim -GH^3e^{5Ht} \,, \tag{6}$$

Dividing by the 3-volume gives a density of about $-GH^6e^{2Ht}$, which rapidly becomes enormous.

Of course this ignores causality. Most of the infrared gravitons needed to maintain $\rho_{\rm IR}$ are produced out of causal contact with one another in different Hubble volumes. The ones in gravitational interaction are those produced within the

same Hubble volume. Since the number of Hubble volumes grows like e^{3Ht}, the rate at which mass is produced within a single Hubble volume is,

$$\frac{dM_1}{dt} \sim H^2 \ . \tag{7}$$

Although most of the newly produced gravitons soon leave the Hubble volume, their gravitational potentials must remain, just as an outside observer continues to feel the gravity of particles that fall into a black hole. The rate at which the Newtonian potential accumulates is therefore,

$$\frac{d\Phi_1}{dt} \sim -\frac{G}{H^{-1}} \frac{dM_1}{dt} \sim -GH^3 \ . \tag{8}$$

Hence the Newtonian gravitational interaction energy density is,

$$\rho(t) \quad \sim \quad \rho_{\mathrm{IR}} \Phi_1(t) \sim -GH^6 Ht \tag{9}$$

$$\sim \quad -\frac{\Lambda}{8\pi G} \cdot (G\Lambda)^2 \cdot Ht \ . \tag{10}$$

When the number of e-foldings Ht becomes large, the fractional rate of change of the gravitational interaction energy is negligible compared with the expansion rate,

$$|\dot{\rho}(t)| \ll H|\rho(t)| \ . \tag{11}$$

It follows from energy conservation,

$$\dot{\rho}(t) = -3H\Big(\rho(t) + p(t)\Big) \ , \tag{12}$$

that the induced pressure must be nearly opposite to the energy density. In other words, back-reaction induces negative vacuum energy.

If inflation persists long enough, the minuscule dimensionless coupling constant in (10) — $(G\Lambda)^2$ — can be overcome by the secular growth in the number of e-foldings, Ht. Of course nonlinear effects become important as well. The physical picture is of inflation ending, rather suddenly, with the universe poised on the verge of gravitational collapse.

Detailed, explicit computations confirm the qualitative analysis presented above, at two loops for pure gravity [15] and at three loops for a massless, minimally coupled ϕ^4 theory in which the scalar self-interaction provides the breaking

mechanism [16]. We worked on the manifold $T^3 \times R$ and used the Schwinger-Keldysh formalism [17] to compute the expectation value of the metric in the presence of a state which is free, Bunch-Davies vacuum at $t = 0$. Since the state is homogeneous and isotropic the expectation value of the metric can be put in the form,

$$\langle \Omega | g_{\mu\nu}(t, \vec{x}) dx^\mu dx^\nu | \Omega \rangle = -dt^2 + e^{2b(t)} d\vec{x} \cdot d\vec{x} \ . \tag{13}$$

We used perturbation theory about the background $b_0(t) = Ht$. There is no secular slowing at one loop because the inflationary production of gravitons is itself a one loop effect and the back-reaction from it must come at a higher order in perturbation theory. Including two loop effects gives the following result for the expansion rate,

$$\dot{b}(t) = H \left\{ 1 - \left(\frac{G\Lambda}{3\pi} \right)^2 \left[\frac{1}{6} (Ht)^2 \right. \right.$$
$$\left. \left. + O(Ht) \right] + O(G^3) \right\} \ . \tag{14}$$

The extra factor of Ht not present in our Newtonian model derives from general relativistic effects having to do with the response to pressure in a locally de Sitter background.

Although we did not compute contributions at three loops and higher we were able to obtain all orders bounds on their time dependence. These bounds show that the two loop result dominates for as long as perturbation theory remains valid [18]. Of course one cannot trust this analysis when the effect becomes of order one. The correct statement is that back-reaction slows inflation by an amount that eventually becomes nonperturbatively large. We have some ideas [19] about how to evolve beyond this point but no firm results as yet.

It is highly significant that this model of inflation contains only a single free parameter, the dimensionless product of Newton's constant and the cosmological constant, $G\Lambda$. This means that the model makes unique and testable predictions in a way that generic scalar-driven inflation can never do. In fact the model's predictions for the CMBR parameters A_S, r, n_S and n_T have recently been worked out [20]. The scalar amplitude A_S sets $G\Lambda$, but the other three are legiti-

mate predictions of the model. They are in agreement with the latest data and will be subject to much closer experimental scrutiny from the precision satellite probes which are due to be flown in the coming decade. These may be the first observations of quantum gravitational phenomena!

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